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**A SPECIALIZED MODEL FOR ANALYSIS OF CREEP-RUPTURE DATA
BY THE MINIMUM COMMITMENT, STATION-FUNCTION APPROACH**

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INTRODUCTION

The need for correlating creep-rupture data and for extrapolation of such data to long rupture times has been treated in recent years by the use of "time-temperature parameters." A large number of such parameters have been proposed, and although many investigations have been conducted to determine their relative ability to perform this correlation and extrapolation, there is no universally accepted procedure for selecting an optimized parameter or for determining the constants involved in such an optimized parametric representation.

In a recent report Manson (ref. 1) has suggested a method which bypasses the subjective aspects of parameter selection, introducing a so-called "minimum-commitment, station-function approach." In this approach the hypothesized time-temperature-stress relation is taken in sufficiently general form to include all commonly used parameters, so that each has an equal chance to emerge as the proper one. The functional forms involved in the relation are not taken in analytical form; rather they are defined as "station-functions" - their numerical values at selected station values of the independent variable. Using station functions not only avoids "forcing" the pattern of the data, but provides an incidental benefit in avoiding illconditioning of the system of resulting equations, which are computer-solved for the optimum data representation. This feature also contributes to the objectiveness of the method, and gives each time-temperature parameter consistent with the model an equal chance to demonstrate itself as the correct one.

In reference 1 the model equation used was in the form

$$F(\log t) + P(T) = G(\log \sigma) \quad (1)$$

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where F is a function of the logarithm of rupture time, t ; P is a function of temperature; and G is a function of stress. Further study has led to an improved model in the form

$$F(T) \log t + P(T) = G(\log \sigma) \quad (2)$$

Here F and P are both functions of temperature, but the time function is replaced by the term $\log t$, which simplifies the extrapolation process to mere substitution of larger values of $\log t$, rather than requiring an extrapolation of the numerically-defined function $F(\log t)$.

Equation (2) is still general enough to embrace all commonly used time-temperature parameters; in fact it is more general than is really required for a first formulation. Therefore, a specialized case is examined first, in which $F(T) = 1 + A P(T)$, resulting in a relation

$$\log t + A P(T) \log t + P(T) = G(\log \sigma) \quad (3)$$

Here A is a constant. It can be shown that this equation not only includes all the commonly used time-temperature parameters, but that even in its specialized form it is general enough to include relations not normally included in the common time-temperature parameters.

In this report we shall outline how equation (3) can be applied to a set of experimental data. Use of the more general form given by equation (2) is left for a later study. The report is presented in the form of a series of narrative figures which are in the main self-explanatory and require only minor comment, which follows.

DISCUSSION

Figure 1 identifies the features regarded as significant in what we have termed the "minimum-commitment, station-function approach." Figure 2 shows the three forms of minimum-commitment formulations that can be used, emphasizing equation (3) which is treated in detail in this report. It is stated

without proof in figure 2 that the function P can be assigned an arbitrary value at any temperature of choice. Specifically, it may be made equal to zero along any one of the available isothermals. That equation (2) is valid for the commonly used Larson-Miller parameter is demonstrated in figure 3, where it is also shown that a function P exists for which $P = 0$ at any arbitrary temperature. The treatment shown in figure 3 for the Larson-Miller parameter can be applied in a similar manner to any of the other commonly used time-temperature parameters.

Table I and figure 4 show the isothermal input data for a material under current study by the NATO AGARD Committee. The data points are estimated, but the set serves well for illustration. Smoothed isothermal plots are drawn through the data; in general such curves can be drawn by eye or by a least squares fit of a second-degree polynomial through the available data. These smoothed curves, of course, are used only for interpolation in the experimentally established rupture time range. The figure also shows that P stations are chosen at each available isothermal, and that G stations are chosen at nine stress values embracing the experimental stress range.

Equations can be derived either from raw data points or from net points chosen at time intervals of $1/4$ log cycle along the smooth isothermals drawn through available data. The method for setting up the equation in the form of equation (3) when smoothed isothermals are used is shown in figure 5 for one net point. Here $\log t = -3.25$, $T_3 = 850$ C, therefore $P = P_3$, and $\log \sigma = 1.146$. Since there is no station at $\log \sigma = 1.146$, the value of $G(1.146)$ is stated in terms of the G values at adjacent stations, using quadratic interpolation.

Figure 6 lists the sources from which the final set of equations is derived. The principal source is the data - either raw data points, or points derived from the faired isothermals. If the main function of the analysis is to extrapolate an isothermal, the equations associated with points derived from that isothermal may be entered into the analysis several times in order to "force" the final correlation to conform best along the isothermal to be extrapolated (usually at some sacrifice to conformity with the experimental data at another

isothermal). Thus, if several isothermals are to be extrapolated, several analyses will be made, each one emphasizing one isothermal. If, however, only a correlation is the main objective, then a single analysis is made; no isothermal is used a multiple number of times. Figure 6 also shows that some of the equations of the system are derived from considerations of continuity of the station-function curves; usually only the curves of $G(\log \sigma)$ versus $\log \sigma$. Such continuity equations are not necessary if there is considerable overlap in range of stress covered by the successive isothermals. When the overlap is very limited the continuity equations are introduced. One form that such a relation can take is

$$G_n - 4G_{n+1} + 6G_{n+2} - 4G_{n+3} + G_{n+4} = 0 \quad (4)$$

Here G_n to G_{n+4} are five successive station function values of equally spaced values of $\log \sigma$. The equation states that the slope at the mid value G_{n+2} is the same whether point G_{n+2} is considered on the quadratic through G_n, G_{n+1}, G_{n+2} or on the quadratic through $G_{n+2}, G_{n+3}, G_{n+4}$. Continuity equations may be entered once or more than once, depending on the degree of stress overlap and on the limit of the number of equations that the available computer can handle.

Once the system of equations is formulated, they can be solved by least squares methods. Unfortunately, when formulated using equation (3) as a model, the resulting equations are non-linear because A is an unknown, as are the values of P which it multiplies. Figure 7 shows two methods of solution. Values of A can be scanned over the practical range of 0.1 to -0.2. The value of A which results in the smallest standard deviation between input data points and their predicted values is taken as the best value, together with its associated values of $P(T)$ and $G(\log \sigma)$. Alternatively, a method of successive approximations is used, as also shown in figure 7. The first approximation is obtained by setting $A = 0$, resulting in a linear set of equations, from which $P_1(T)$ may be determined. This first approximation of $P(T)$ is then used as a known function in that part of the equation which is

multiplied by the unknown A , whereas $P(T)$ is left as an unknown where it stands alone linearly. A new value of $P_2(T)$ is determined, which is then treated in the same way to get a third approximation. The process is repeated until successive calculations cause no change between respective $P(T)$ values (within the third significant figure), at which time the solution is regarded as converged. Of course, the entire process is completely computerized, and the computer output provides graphical displays of all functions and the back-computed isothermals from the station functions derived from the solution.

Figure 8 shows the results of the application of the foregoing procedure to the NATO data set of figure 4. The analysis presumes that data are available up to 3200 hours rupture time. Because the number of "raw" data points in the rupture time range below 3200 hours is quite limited, the analyses is performed on only the "faired" curves through the data; however, if each isothermal were established by a more exclusive series of data points, the analyses could equally well be made by using the raw data points. The analysis was made using both the A-scan method and the successive approximations method. It is seen that both methods indicate an optimum value of A close to -0.1 ; the exact value actually is not critical since each value of A carries with it different $P(T)$ and $G(\log \sigma)$ functions which compensate for the difference in A resulting in very similar correlations and extrapolations over the range of values of A . Therefore, the rounded value of $A = -0.1$ was regarded as satisfactory, and was used for the calculation of the isothermals shown in figure 9.

In this case all isothermals were treated equally; that is no multiple use of data was used for any one isothermal. If any one isothermal had been regarded as well established experimentally, then the analysis for that isothermal could have been made by using the points on that isothermal more than once.

Figures 10 and 11 show the $P(T)$ and $G(\log \sigma)$ functions associated with the value of $A = -0.1$. The computer program provides plots of all functions, those of figures 10 and 11, required in the analysis.

CONCLUDING REMARKS

Treatment of data sets by application of equation (3) is now completely computerized. When applied to artificial data sets, such as those conforming to the Larson-Miller, Manson-Haferd, or Orr-Sherby-Dorn parameters the extrapolations (by a time factor of 20) were essentially identical to those determined from the analytical relations used in generating the short time data. Thus the method will produce the correct predictions if the basic data conform to one of the commonly used time-temperature parameters, and when the input data is plentiful and given to a high degree of accuracy. Obviously most experimental data sets will fall short in quantity and quality of the type of data points that lend themselves to an ideal analysis. Much yet remains to be done to determine how best to use the method and what its limitations are.

Figure 12 outlines some areas of possible joint endeavor among groups interested in the problem of correlation and extrapolation of stress-rupture data. One of the main goals would be for the involved technical community to arrive at a common viewpoint regarding validity of this or any other procedure for correlating creep rupture data. This can best be accomplished by actually trying out the procedure on individual data sets within each investigator's experience. From direct experience it will become evident how such data might have better served the purpose if they incorporated certain desirable features, and therefore how data should be generated in the future. Perhaps it will become evident that certain constants or functional forms can be assumed to be the same for all materials or for certain classes of materials. The actual generation of data, individually or collectively could also be a ground for cooperative effort. How data-scatter, heat-to-heat variations, material instabilities, and other complicating factors are to be treated also requires joint investigation. As an ultimate goal, we can hope to achieve a common agreement on how best to generate data and how best to analyze such data - a set of "standards" we can all accept and follow.

REFERENCE

Manson, S. S.: Time-Temperature Parameters-A Re-Evaluation and Some New Approaches, ASM Publication No. D8-100.

TABLE I. - RAW DATA SUPPLIED BY THE
NATO AGARD COMMITTEE

Number	Temperature	Stress, kg/mm ²	Time, hr	Log, stress	Log, time
1	750.	72.1	20.0	1.857	1.301
2	750.	38.7	1000.0	1.587	3.000
3	750.	28.4	7000.0	1.453	3.845
4	750.	20.8	30000.0	1.318	4.477
5	800.	38.7	100.0	1.587	2.000
6	800.	28.4	800.0	1.453	2.903
7	800.	20.8	3000.0	1.318	3.477
8	800.	15.3	10000.0	1.184	4.000
9	850.	28.4	50.0	1.453	1.698
10	850.	20.8	400.0	1.318	2.602
11	850.	15.3	1000.0	1.184	3.000
12	850.	11.2	5000.0	1.049	3.698
13	850.	8.2	10000.0	.913	4.000
14	850.	6.0	25000.0	.778	4.397
15	900.	20.8	50.0	1.318	1.698
16	900.	15.3	150.0	1.184	2.176
17	900.	11.2	600.0	1.049	2.778
18	900.	8.2	1500.0	.913	3.176
19	900.	6.0	3000.0	.778	3.477
20	900.	4.4	7000.0	.643	3.845
21	900.	3.2	10000.0	.505	4.000
22	900.	2.4	20000.0	.380	4.301
23	950.	11.2	40.0	1.049	1.602
24	950.	8.2	200.0	.913	2.301
25	950.	6.0	800.0	.778	2.903
26	950.	4.4	1500.0	.643	3.176
27	950.	3.2	2000.0	.505	3.301
28	950.	2.4	5000.0	.380	3.698
29	950.	1.7	10000.0	.230	4.000

TABLE II. - SMOOTHED DATA READ FROM

ISOTHERMALS OF NATO DATA

Number	Temperature	Stress, kg/mm ²	Time, hr	Log, stress	Log, time
1	750.	67.50	31.6	1.829	1.50
2	750.	63.00	56.2	1.799	1.75
3	750.	58.00	100.0	1.763	2.00
4	750.	53.50	177.8	1.728	2.25
5	750.	49.00	316.2	1.690	2.50
6	750.	45.00	562.3	1.653	2.75
7	750.	40.50	1000.0	1.607	3.00
8	750.	36.50	1778.0	1.562	3.25
9	750.	33.00	3162.0	1.518	3.50
10	750.	30.00	5623.0	1.477	3.75
11	750.	26.50	10000.0	1.423	4.00
12	750.	23.00	17780.0	1.361	4.25
13	750.	20.00	31620.0	1.301	4.50
14	800.	38.50	100.0	1.585	2.00
15	800.	36.00	177.8	1.556	2.25
16	800.	33.00	316.2	1.518	2.50
17	800.	30.00	562.3	1.477	2.75
18	800.	27.00	1000.0	1.431	3.00
19	800.	24.00	1778.0	1.380	3.25
20	800.	21.00	3162.0	1.322	3.50
21	800.	18.00	5623.0	1.255	3.75
22	800.	15.00	10000.0	1.176	4.00
23	850.	28.50	56.2	1.454	1.75
24	850.	26.00	100.0	1.414	2.00
25	850.	23.50	177.8	1.371	2.25
26	850.	21.00	316.2	1.322	2.50
27	850.	18.50	562.3	1.267	2.75
28	850.	16.00	1000.0	1.204	3.00
29	850.	14.00	1778.0	1.146	3.25
30	850.	12.20	3162.0	1.086	3.50
31	850.	10.30	5623.0	1.012	3.75
32	850.	8.60	10000.0	.934	4.00
33	850.	7.00	17780.0	.845	4.25
34	900.	20.00	56.2	1.301	1.75
35	900.	17.50	100.0	1.243	2.00
36	900.	15.00	177.8	1.176	2.25
37	900.	12.80	316.2	1.107	2.50
38	900.	10.80	562.3	1.033	2.75
39	900.	9.20	1000.0	.963	3.00
40	900.	7.50	1778.0	.875	3.25
41	900.	6.20	3162.0	.792	3.50
42	900.	4.80	5623.0	.681	3.75
43	900.	3.65	10000.0	.562	4.00
44	900.	2.60	17780.0	.414	4.25
45	950.	10.50	56.2	1.021	1.75
46	950.	9.50	100.0	.977	2.00
47	950.	8.40	177.8	.924	2.25
48	950.	7.30	316.2	.863	2.50
49	950.	6.20	562.3	.792	2.75
50	950.	5.10	1000.0	.707	3.00
51	950.	4.10	1778.0	.612	3.25
52	950.	3.15	3162.0	.498	3.50
53	950.	2.30	5623.0	.361	3.75
54	950.	1.65	10000.0	.217	4.00

Note: Only data $\leq \log t = 3.5$ were used to predict isothermals shown in figure 9.

BASIS OF THE MINIMUM-COMMITMENT STATION-FUNCTION APPROACH

- I. CHOICE OF GENERAL MODEL TIME-TEMPERATURE-STRESS RELATION EMBRACING AT LEAST ALL COMMONLY USED PARAMETRIC FORMS. "GROWTH POTENTIAL" OF EQUATION FORM DESIRABLE.
- II. EXPRESSION OF FUNCTIONS INVOLVED IN MODEL EQUATION BY NUMERICAL VALUES OF FUNCTION AT SPECIFIED "STATIONS" ALONG PERTINENT VARIABLE SCALE. INTERPOLATION FORMULAS USED FOR INTERMEDIATE VALUES OF INDEPENDENT VARIABLE.
- III. REPRESENTATION OF AVAILABLE EXPERIMENTAL DATA AS SERIES OF RELATIONS AMONG TIME, TEMPERATURE, AND STRESS CONFORMING TO MODEL EQUATION.
- IV. SOLUTION OF EQUATIONS BY COMPUTERIZED LEAST-SQUARES PROCEDURES.

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Fig. 1

FUNCTIONAL RELATIONS CONFORMING WITH COMMONLY USED TIME TEMPERATURE PARAMETERS

I. PREVIOUSLY USED:

$$F(\log t) + P(T) = G(\log \sigma) \quad (1)$$

II. IMPROVED FORM:

$$F(T) \log t + P(T) = G(\log \sigma) \quad (2)$$

III. SPECIALIZED FORM OF EQUATION (2) WHEREIN $F = 1 + AP$:

$$\log t + AP(T) \log t + P(T) = G(\log \sigma) \quad (3)$$

$\log t$ = LOG RUPTURE TIME

A = CONSTANT

$P(T)$ = FUNCTION OF TEMPERATURE, HANDLED AS A STATION FUNCTION

$G(\log \sigma)$ = FUNCTION OF STRESS, HANDLED AS STATION FUNCTION

THE FUNCTION $P(T)$ CAN ARBITRARILY BE ASSIGNED A ZERO VALUE AT ANY SINGLE SELECTED TEMPERATURE.

Fig. 2

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EXAMPLE OF EXPRESSION OF COMMONLY USED TIME TEMPERATURE PARAMETER IN SPECIALIZED FORM

CONSIDER LARSON-MILLER PARAMETER

$$(T + 460)(20 + \log t)$$

PARAMETRIC VALIDITY IS NOT ALTERED BY MULTIPLYING BY CONSTANT OR ADDING A CONSTANT. THEREFORE PARAMETER CAN EQUALLY WELL BE EXPRESSED AS $M(T + 460)(20 + \log t) + N$

CHOOSING $N = -20$ AND $M = \frac{1}{T + 460}$ WHERE T_0 IS ANY TEMPERATURE

WE CAN REWRITE PARAMETER AS

$$\log t + \frac{1}{20} \left[20 \left(\frac{T + 460}{T_0 + 460} - 1 \right) \right] \log t + \left[20 \left(\frac{T + 460}{T_0 + 460} - 1 \right) \right]$$

WHICH IS IN FORM: $\log t + A P(T) \log t + P(T)$

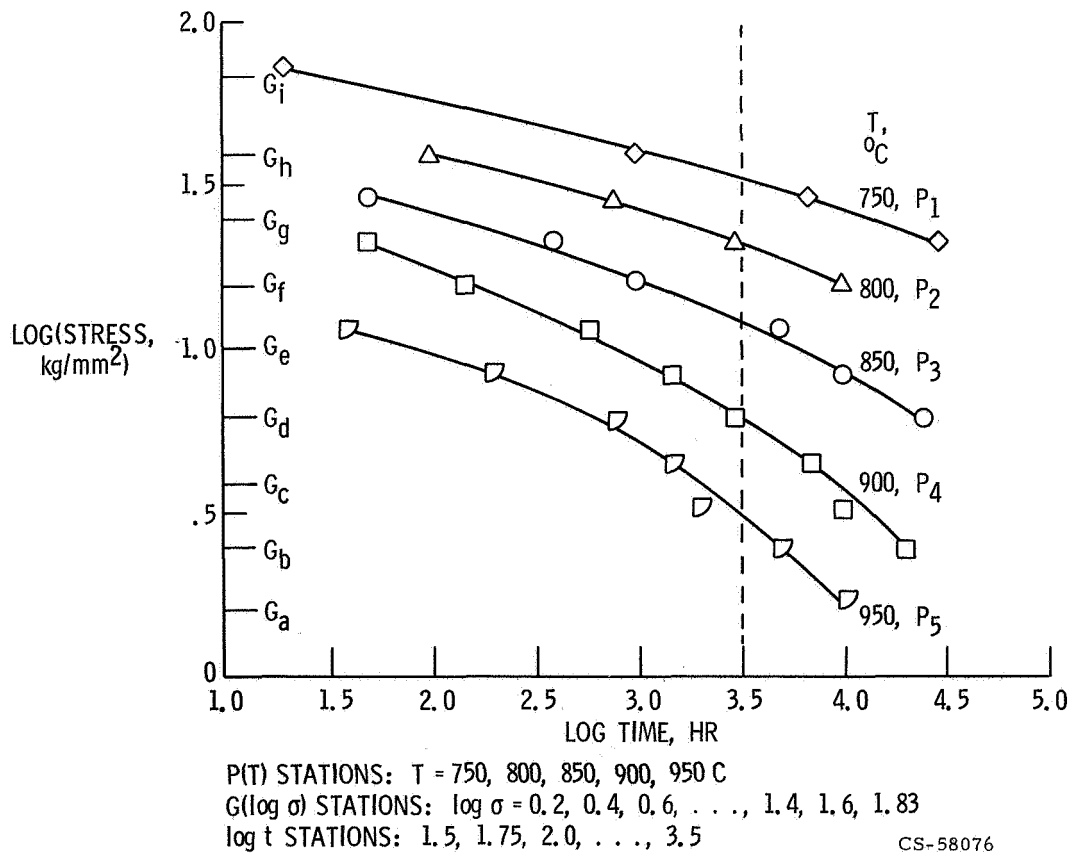
WHERE $A = 1/20$

$$P(T) = \frac{T + 460}{T_0 + 460} - 1 \quad \text{NOTE THAT } P(T) \text{ IS ZERO AT } T = T_0$$

Fig. 3

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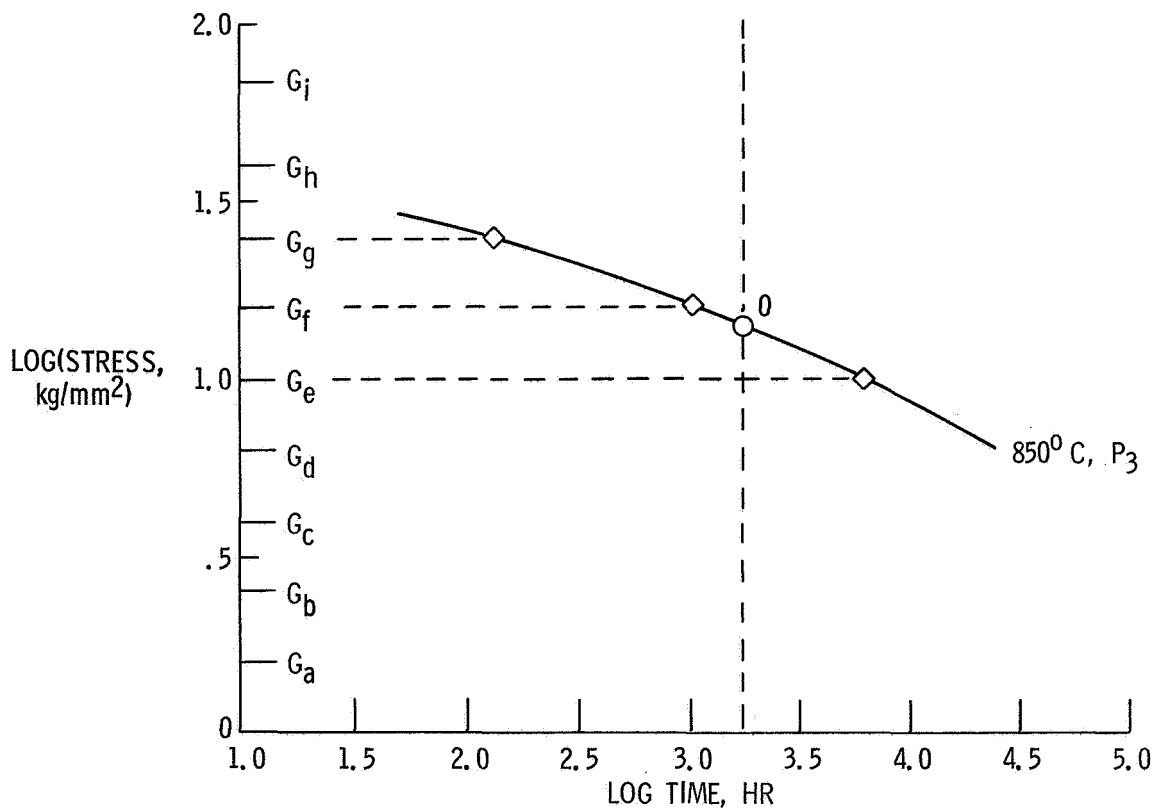
SET-UP OF STATIONS FOR NATO SAMPLE DATA SET



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Fig. 4

TYPICAL FORMULATION OF EQUATION FOR ONE DATA POINT



AT POINT 0

$\log t = 3.25, P = P_3$

$\log \sigma = 1.146$

THEREFORE $G \approx 0.171 G_e + 0.927 G_f - 0.098 G_g$

APPLYING EQUATION $\log t + AP(T)\log t + P(T) = G(\log \sigma)$

$3.25 + 3.25 AP_2 + P_2 = 0.171 G_e + 0.927 G_f - 0.098 G_g$

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Fig. 5

TYPES OF EQUATIONS USED IN FORMULATION

- I. INDIVIDUAL DATA POINTS, ONE FOR EACH POINT, USING "RAW" DATA OR NET POINTS FROM FAIRED CURVES.
- II. MULTIPLE USE OF DATA POINTS ALONG ISOTHERMAL TO BE EXTRAPOLATED (THREE TO TEN TIMES, DEPENDING ON LIMITS OF COMPUTER AVAILABLE).
- III. "CONTINUITY" EQUATIONS TO INSURE SMOOTHNESS OF CURVES. (MAY NOT BE NECESSARY WHEN SMOOTHED ISOTHERMALS ARE USED FOR NET POINTS.)

Fig. 6

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SOLUTION OF SET OF EQUATIONS

FOR SPECIALIZED MODEL, SOLUTION IS COMPLICATED BY NONLINEAR FORM RESULTING FROM PRODUCT OF UNKNOWN A AND P . CAN PROCEED BY:

I. TRIAL AND ERROR:

SCAN SERIES OF VALUES OF A ; e.g., $A = 0.1, 0.05, 0, -0.05, -0.1, -0.12$, ETC. LOWEST STANDARD DEVIATION IDENTIFIES OPTIMUM A AND ASSOCIATED P AND G FUNCTIONS.

II. SUCCESSIVE APPROXIMATIONS:

A. START WITH $A = 0$, EQUATION BECOMES

$$\log t + P_1(T) = G_1(\log \sigma)$$

WHICH IS LINEAR SET AND CAN READILY BE SOLVED. GET SECOND APPROXIMATION $P_2(T)$ BY USING $P_1(T)$ ONLY IN NONLINEAR PORTION OF EQUATION.

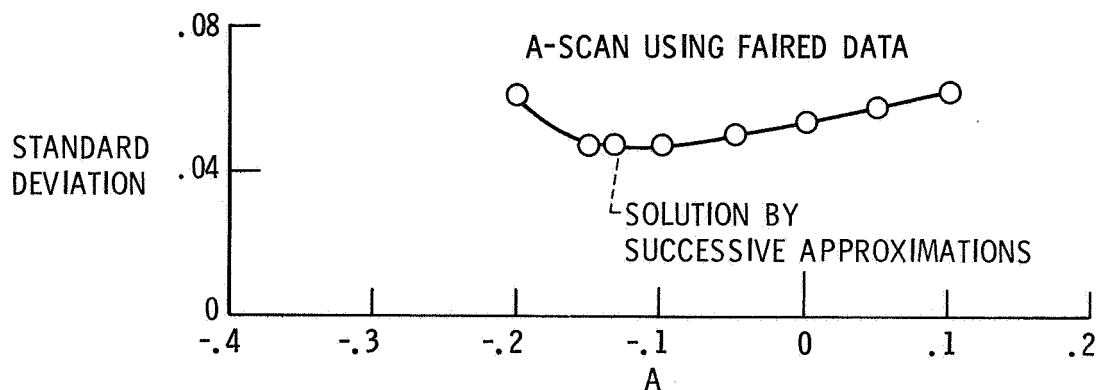
$$\log t + AP_1(T) \log t + P_2(T) = G_2(\log \sigma)$$

SOLVE LINEAR SET FOR $P_2(T)$ AND REPEAT PROCEDURE UNTIL SOLUTION CONVERGES BY SHOWING NO CHANGE IN THIRD SIGNIFICANT FIGURE IN SUCCESSIVE CALCULATIONS. PROCESS IS COMPLETELY COMPUTERIZED. GRAPHICAL DISPLAY OF SOLUTION IS ALSO COMPUTERIZED.

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Fig. 7

EXAMPLE: APPLICATION TO NATO HYPOTHETICAL DATA SET



BOTH APPROACHES SHOW VALUE OF A IN VICINITY OF -0.1 . THE EXACT VALUE OF A IS NOT CRITICAL IN AFFECTING EXTRAPOLATIONS.

Fig. 8

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EXTRAPOLATIONS FOR NATO HYPOTHETICAL DATA SET

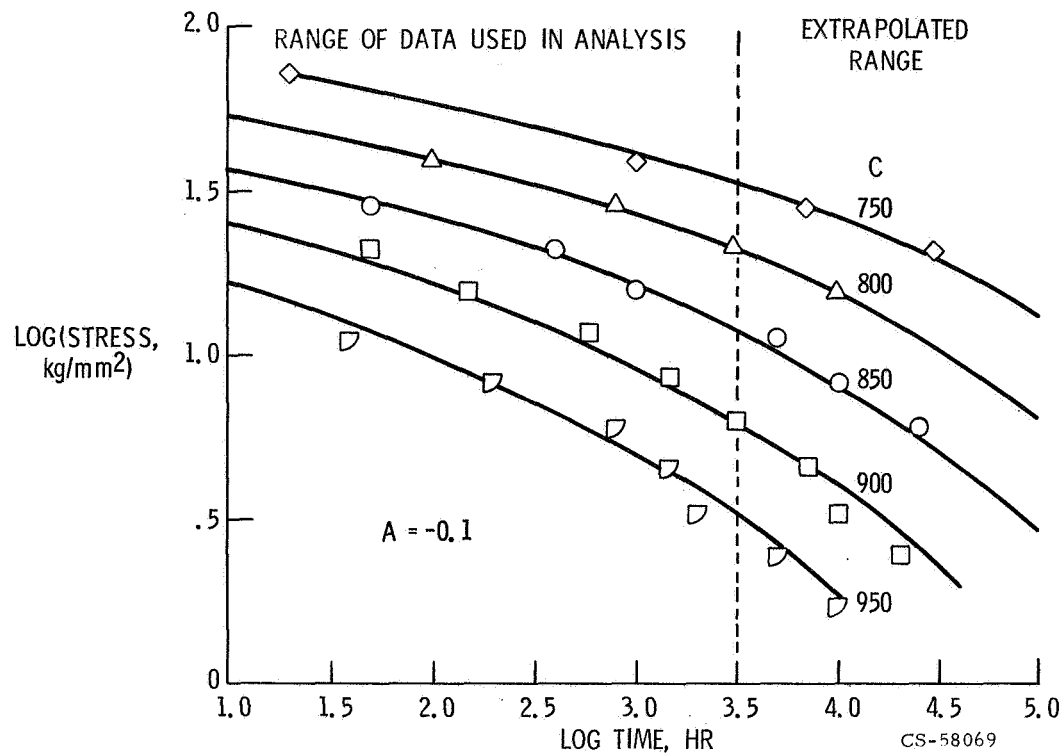


Fig. 9

TEMPERATURE FUNCTION

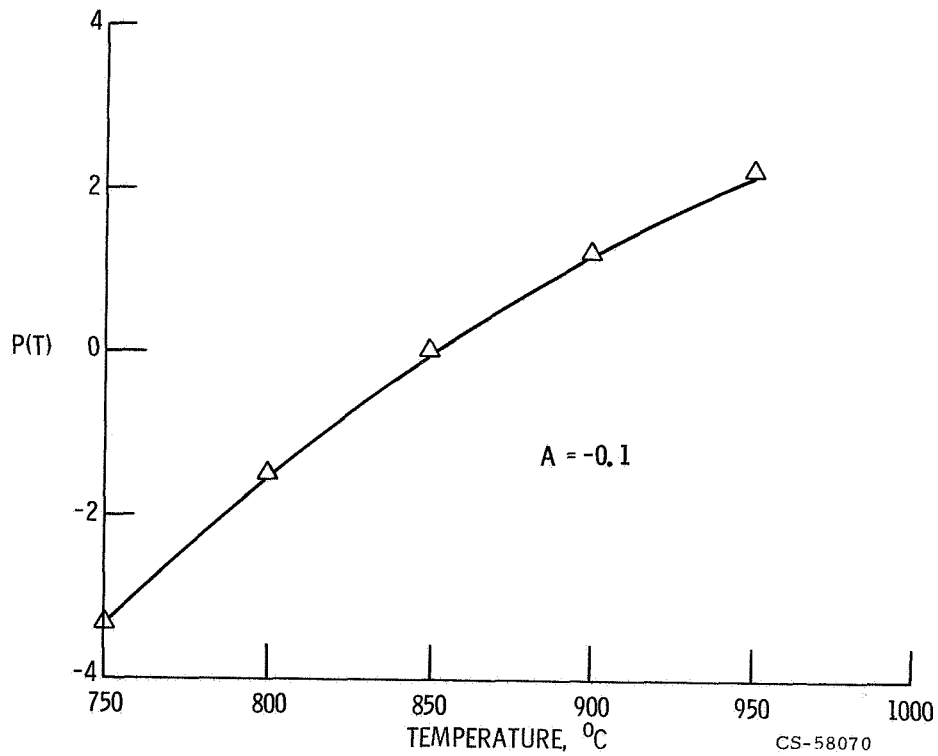


Fig. 10

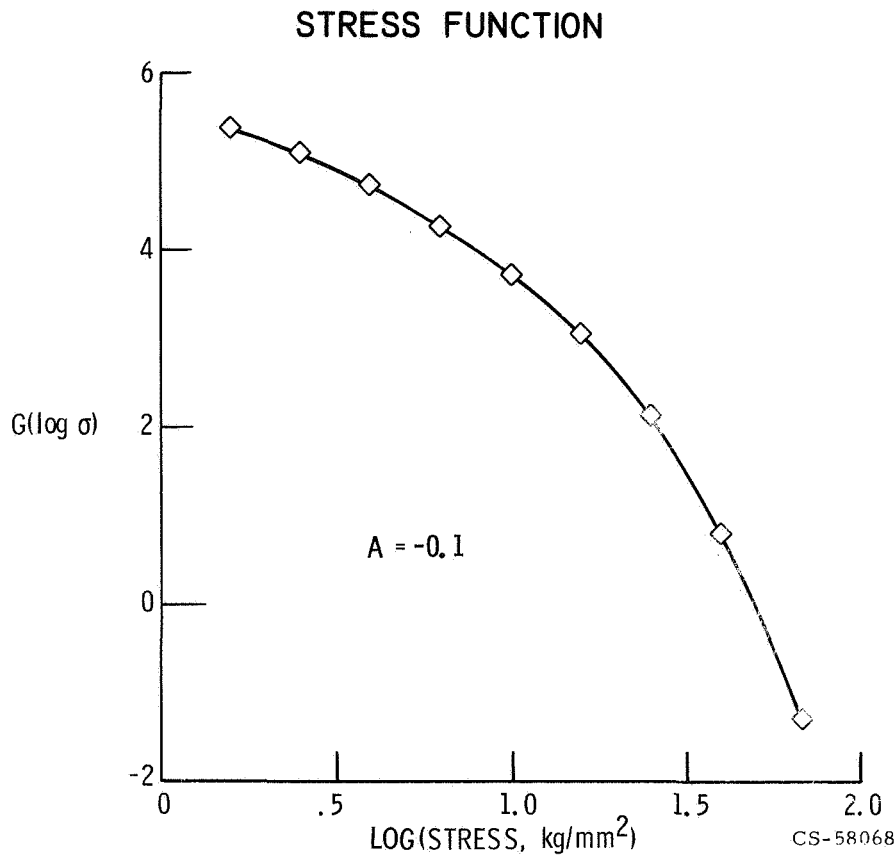


Fig. 11

POSSIBLE AREAS FOR JOINT INQUIRY

1. COMPARE THIS METHOD WITH COMMONLY USED TIME TEMPERATURE PARAMETERS.
2. EVALUATE METHOD ON DATA SETS AVAILABLE TO MEMBERS IN ORDER TO ESTABLISH CONFIDENCE, OR TO REFLECT PROBLEMS OF PROCEDURE.
3. DETERMINE CRITERIA FOR OPTIMIZING THE GENERATION OF NEW DATA FOR USE IN METHOD.
4. ESTABLISH WHETHER THERE ARE CONSTANTS OR FUNCTIONAL FORMS THAT CAN BE "UNIVERSALIZED" FOR ALL MATERIALS OR FOR CIRCUMSCRIBED CLASSES OF MATERIALS.
5. STUDY EFFECT OF MATERIAL INSTABILITIES OR OTHER METALLURGICAL COMPLICATIONS ON VALIDITY OF THIS APPROACH.
6. STUDY PROBLEMS ASSOCIATED WITH DATA SCATTER, LOT-TO-LOT VARIATIONS, ACCEPTANCE TESTING, AND OTHER PRACTICAL FACTORS.
7. EVALUATE POSSIBILITY OF SETTING "STANDARDS."

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Fig. 12